行政院國家科學委員會專題研究計畫成果報告

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* 行 進 波 之 函 數 理 論 法 *

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國科會助理：喻芝健
國科會助理：劉建德
共同主持人：

整合型計劃：總計劃主持人：
子計劃主持人：

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執行單位：淡江大學數學系
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中文摘要:

在此報告中我們研究由無限個蔡氏電路耦合所形成的理想化的偏微分方程之行進波解的存在性. 我們先利用變數變換的方法將原偏微分方程轉換成一邊界值問題, 再利用上近似解與下近似解的方法得到當參數在某些值域時此邊界值微分方程有異源解. 因而我們可以得到耦合蔡式電路有任意速度的行進波解的結論.

關鍵詞：

上近似解, 下近似解, Nagumo 條件, 行進波解, 異源解.

英文摘要:

In this work, we study the existence of heteroclinic orbits for a ordinary differential equations which arise from a one-dimensional array of Chua's circuits. By using the upper and lower solutions method and a zero-order approximation , we show that for a certain set of parameters we have the existence of traveling wave solutions for the idealized system for some given speed.

關鍵詞：

upper solution, lower solution, Nagumo condition, traveling solution, heteroclinic orbit.

1 Summary

In this work we will study the partial differential equation which had been derived by idealizing the infinitely coupled chua's circuits.

$$\begin{aligned}
\frac{\partial U}{\partial t} &= \alpha(Z - f(U)) + D\frac{\partial^2 U}{\partial x^2}, \quad D > 0, \\
\frac{\partial Z}{\partial t} &= U - Z + W, \\
\frac{\partial W}{\partial t} &= -\beta Z.
\end{aligned} \tag{1.1}$$

where

$$f(u) = \begin{cases} m_0 u, & u \leq \bar{u}_1, \\ -m_1(u - \bar{u}), & \bar{u}_1 < u \leq \bar{u}_2, \\ m_2(u-1), & u > \bar{u}_2. \end{cases}$$

Mainly, we are interesting in the existence of traveling solution of equation (1.1). Let c be the wave speed of the traveling wave solution. By introducing a moving coordinate $t'' = \frac{t'}{-c} = \frac{1}{-c}(x - ct)$ and setting

$$U(t,x) = u(\frac{x}{-c} + t), \quad Z(t,x) = z(\frac{x}{-c} + t), \quad W(t,x) = w(\frac{x}{-c} + t),$$

we arrive at ordinary differential equations for (u, z, w):

$$\begin{cases} \varepsilon\ddot{u} = \dot{u} - \alpha(z - f(u)), \\ \dot{z} = u - z + w, \\ \dot{w} = -\beta z \end{cases} \tag{1.2}$$

with the boundary conditions

$$\lim_{t\to-\infty}(u, \dot{u}, z, w) = (0,0,0,0), \qquad \lim_{t\to+\infty}(u, \dot{u}, z, w) = (1,0,0,-1), \tag{1.3}$$

where $\varepsilon = \dfrac{D}{c^2}$.

To show that the system (1.2) with boundary condition (1.3) has a solution for a certain set of parameter values. We will need the following existence theorems for a certain class of boundary value problem.

Let $\mathcal{T} : C^0([a,b],\mathbb{R}) \to C^2([a,b],\mathbb{R})$ be a bounded linear operator and $f : [a,b] \times \mathbb{R} \times \mathbb{R} \times C^0([a,b],\mathbb{R}) \to \mathbb{R}$ be continuous.

Theorem 1.1 : If $f(t,y,z,[\mathcal{T}\xi])$ is bounded on $[a,b] \times \mathbb{R} \times \mathbb{R} \times C^1([a,b],\mathbb{R})$. Then for and $A, B \in \mathbb{R}$, the boundary value problem

$$y'' = f(t,y,y',[\mathcal{T}y]),$$
$$y(a) = A, \quad y(b) = B$$

has a solution.

Definition 1.2 : Let $\underline{\omega}$ and $\overline{\omega}$ be continuous and piecewise C^2 functions with $\underline{\omega}(t) \leq \overline{\omega}(t)$ on $[a,b]$. That is, there is a finite partition $\{t_i\}$, $i = 0,\cdots,n$, of $[a,b]$ with $a = t_0 < t_1 < t_2 < \cdots < t_n = b$, such that on each closed subinterval $[t_{i-1},t_i]$, $\underline{\omega}$ and $\overline{\omega}$ are twice continuously differentiable. At the partition points t_{i-1} and t_i, the right and left-handed derivatives satisfy the following:

$$\underline{\omega}'(t_i^-) < \underline{\omega}'(t_i^+), \quad \overline{\omega}'(t_i^-) > \overline{\omega}'(t_i^+), \quad i = 1,2,\cdots,n-1,$$

In addition, if on each open subinterval (t_{i-1},t_i) the following inequalities are satisfied for any $\xi(t) \in C^0([a,b]$ satisfying $\underline{\omega}(t) \leq \xi(t) \leq \overline{\omega}(t)$,

$$\underline{\omega}''(t) \geq f(t,\underline{\omega}(t),\underline{\omega}'(t),[\mathcal{T}\xi](t)),$$
$$\overline{\omega}''(t) \leq f(t,\overline{\omega}(t),\overline{\omega}'(t),[\mathcal{T}\xi](t))$$

then $\underline{\omega}(t)$ and $\overline{\omega}(t)$ are called the piecewise C^2 lower and upper solution, respectively, of the differential equation

$$y'' = f(t,y,y',[\mathcal{T}\xi(t)]) \tag{1.4}$$

Definition 1.3: Let $\underline{\omega}$, $\overline{\omega} \in C^0([a,b],\mathbb{R})$ with $\underline{\omega}(t) \leq \overline{\omega}(t)$ on $[a,b]$ and $h \in C^0(\mathbb{R}^+,\mathbb{R}^+)$ satisfy

$$\int_\lambda^\infty \frac{\mathrm{s}ds}{h(s)} > \max_{t\in[a,b]} \overline{\omega}(t) - \min_{t\in[a,b]} \underline{\omega}(t),$$

where

$$\lambda(b-a) = \max\{|\underline{\omega}(a) - \overline{\omega}(b)|, |\underline{\omega}(b) - \overline{\omega}(a)|\}.$$

The differential equation (1.4) is said to satisfy a Nagumo condition on $[a,b]$ with respect to the pair $\underline{\omega}, \overline{\omega}$ if the following inequality is satisfied for any $t \in [a,b]$, $\underline{\omega}(t) \leq u \leq \overline{\omega}(t)$, $v \in \mathbb{R}$ and any $\xi \in C^0([a,b],\mathbb{R})$ with $\underline{\omega}(t) \leq \xi(t) \leq \overline{\omega}(t)$,

$$|f(t,u,v,[\mathcal{T}\xi](t))| \leq h(|v|).$$

Theorem 1.4: Suppose that $\underline{\omega}$ and $\overline{\omega}$ are piecewise C^2 lower and upper solutions of the differential equation

$$y'' = f(t,y,y',[\mathcal{T}\xi](t)),$$

where $f : [a,b] \times \mathbb{R} \times \mathbb{R} \times C^0([a,b],\mathbb{R})$ is continuous and $\mathcal{T} : C^0([a,b],\mathbb{R}) \to C^2([a,b],\mathbb{R})$ is a bounded linear operator . Assume that $y'' = f(t,y,y',[\mathcal{T}\xi](t))$ satisfies a Nagumo condition on $[a,b]$ with respect to the pair $\underline{\omega}$ and $\overline{\omega}$. Then for any A, B with $\underline{\omega}(a) \leq A \leq \overline{\omega}(a)$ and $\underline{\omega}(b) \leq B \leq \overline{\omega}(b)$, the following boundary value problem

$$\begin{aligned} y'' &= f(t,y,y',[\mathcal{T}y]), \\ y(a) &= A, \quad y(b) = B, \end{aligned}$$

has a solution $y(t)$ with $\underline{\omega}(t) \leq y(t) \leq \overline{\omega}(t)$ on $[a,b]$. Moreover, let $N_0 > 0$ be determined by:

$$\int_\lambda^{N_0} \frac{\mathrm{s}ds}{h(s)} = \max_{t\in[a,b]} \overline{\omega}(t) - \min_{t\in[a,b]} \underline{\omega}(t),$$

then $|y'(t)| \leq N_0$, for all $a \leq t \leq b$.

Now let's define the zero order approximation solution (u_0, z_0, w_0) of (1.2) as follows

$$u_0(t) = \begin{cases} u_0^L(t) = e^{\frac{t}{\varepsilon}}, & t \leq 0, \\ u_0^R(t) = 1, & t > 0. \end{cases}$$

$$z_0(t) = \begin{cases} z_0^L(t) = \frac{\varepsilon}{1+\varepsilon+\beta\varepsilon^2} e^{\frac{t}{\varepsilon}}, & t \leq 0, \\ z_0^R(t) = [\frac{\varepsilon}{1+\varepsilon+\beta\varepsilon^2}\cos(\bar{\beta}t) + (\frac{1}{\bar{\beta}} - \frac{(1+2\beta\varepsilon)\varepsilon}{\bar{\beta}(1+\varepsilon+\beta\varepsilon^2)})\sin(\bar{\beta}t)]e^{-\frac{1}{2}t}, & t > 0, \end{cases}$$

$$w_0(t) = \begin{cases} w_0^L(t) = -\frac{\beta\varepsilon^2}{1+\varepsilon+\beta\varepsilon^2} e^{\frac{t}{\varepsilon}}, & t \leq 0, \\ w_0^R(t) = [\frac{1+\varepsilon}{1+\varepsilon+\beta\varepsilon^2}\cos(\bar{\beta}t) + \frac{1+(1-2\beta)\varepsilon}{2\bar{\beta}(1+\varepsilon+\beta\varepsilon^2)}\sin(\bar{\beta}t)]e^{-\frac{1}{2}t} - 1, & t > 0, \end{cases}$$

where $\bar{\beta} = \sqrt{\beta - \frac{1}{4}}$.

Our main result is as follows.

Theorem 1.5. Consider the following differential equation

$$\begin{cases} \varepsilon\ddot{u} = \dot{u} - \alpha(z - f(u)), \\ \dot{z} = u - z + w, \\ \dot{w} = -\beta z \end{cases}$$

with the boundary conditions

$$\lim_{t\to-\infty}(u, \dot{u}, z, w) = (0, 0, 0, 0), \quad \lim_{t\to+\infty}(u, \dot{u}, z, w) = (1, 0, 0, -1),$$

If $\varepsilon > 0$ is sufficiently small, $\beta > \frac{1}{4}$ and $a > 0$ is sufficiently large , then for any $0 < b < \frac{1}{2}$ the boundary value problem (1.2) (1.3) has solution $(u, \dot{u}, z, w)$.

Moreover, they satisfy the following estimates

$$|(u,z,w) - (u_0^L(t), z_0^L(t), w_0^L(t))| \le Me^{at}, \qquad t \le 0,$$
$$|(u,z,w) - (u_0^R(t), z_0^R(t), w_0^R(t))| \le Me^{-bt}, \qquad t > 0.$$

where some appropriate constant M .

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